Włodzimierz Freda
Akademia Morska w Gdyni

REVISED PARAMETERIZATION OF FOURNIER-FORAND PHASE FUNCTIONS FOR SOUTHERN BALTIC WATERS

Detection and identification of oil substances in sea water can be effective only by using remote optical methods. That entails the need to distinguish impurities from natural constituents of seawater. This calls for knowledge of the optical properties of sea water and especially the phase function that describe the angular distribution of scattered light intensities. Measurements of the phase function in the marine environment are rarely performed. Therefore some analytic form of phase functions are needed. One of such forms that adapts well to the results of measurements, is the Fournier-Forand phase function. The parameters of this function are the slope of the size distribution, and the relative refractive index. Since the determination of the latter one is a problematic task, attempts were made to replace these parameters with other quantities. Freda and Piskozub (2007) proposed to parameterize it with an absorption coefficient and the backscattering ratio. However their parameterization demand to find the solution of equations that can be most easily obtained by numerical methods. For that reason, a present paper proposes a simplified parameterization of Freda and Piskozub and provides a simple formulas connecting the Fournier-Forand parameters to the absorption coefficient.

INTRODUCTION

Detection and identification of oil substances in sea water is an important issue because of the ecological aspects of the increased use of the oceans as a way of transport. This identification can be effective only when using remote optical methods. Using these methods entails the requirement to distinguish impurities from natural constituents of seawater. This calls for knowledge of the optical properties of sea water. Optical properties of the oil contaminants, which may be present in the sea water in the form of oil emulsions were presented before [14-16].

The light scattering phase function is the least known optical property of seawater. It describes the angular distributions of intensities of light scattered in seawater. Moreover the measurements of these functions in a large range of scattering angles are performed in marine waters very rarely. Knowledge of these functions is necessary for calculations of the radiative transfer in sea water. For that reason, the calculations are often used with averaged values of the phase function measured by Petzold more than 40 years ago [17] or some kind of an analytic form of phase function.

One of the analytic phase function, which adapts well to results of measurements is a function proposed by Fournier, Forand, and Jonasz in a series of articles [5], [6] and [7]. It is a mathematical approximation of the light scattering function for a set of scattering particles satisfying the hypotheses of Mie theory, which sizes can be described by the hyperbolic size distribution. The so-called Fournier-Forand phase function (FF) is expressed by two parameters, the real part of refractive index n and the differential slope of the hyperbolic (power law) particle size distribution μ.

1. FOURNIER-FORAND PHASE FUNCTION

The Fournier-Forand phase function was proposed as a product of the single particle phase function $P(\theta, x)$ and the scattering cross section, which was weighted for all sizes:

$$\beta(\theta) = \int_0^\infty \pi r_p^{\,2} Q_s(x) P(\theta, x) f(r_p) dr_p, \tag{1}$$

where r_p is the radius of scattering particles, x is the dimensionless particle size parameter $x = 2\pi r_p/\lambda$, and λ is a wavelength of light, $Q_s(x)$ is the efficiency of scattering from a single particle, defined as the ratio of the scattering cross section to the geometric cross-section of the particles, and $f(r_p)$ is the distribution of particle sizes, called the Junge's size distribution:

$$f(r_p) = \frac{C_J}{r_p^{\,\mu}}, \tag{2}$$

where C_J is a constant value of the Junge's size distribution. The efficiency of scattering $Q_s(x)$ was approximated according to the anomalous diffraction theory for scattering by:

$$Q_s(x) = \frac{\rho^2/2}{1+\rho^2/4}, \tag{3}$$

where $\rho = 2(n-1)x$ is the difference in phases between the scattered light and the central ray, while n is the real part of relative refractive index.

Scattering phase function for a single particle $P(\theta, x)$ in both the diffraction theory and the Rayleigh-Gans theory contains complex Bessel functions. Fournier and Forand decided to approximate it with relationship:

$$P(\theta, x) = \frac{1+4x^2/3}{4\pi\left(1+u^2x^2/3\right)}, \tag{4}$$

where: $u = 2\sin(\theta/2)$. Integration of equation (1) gave the scattering coefficient b:

$$b(\lambda,\mu,n) = C_J \frac{\pi}{\cos(\pi\mu/2)} \left(\frac{2\pi(n-1)}{\lambda} \right)^{\mu-3} . \tag{5}$$

While substituting equation (3) and (4) to equation (1) one can obtain a form of volume scattering function, which when divided by the scattering coefficient gives the Fournier-Forand scattering phase function:

$$\tilde{\beta}(\theta,\mu,n) = \frac{1}{4\pi(1-\delta)^2 \delta^\nu} \left[\left[\nu(1-\delta) - (1-\delta^\nu) \right] + \frac{4}{u^2} \left[\delta(1-\delta^\nu) - \nu(1-\delta) \right] \right] + \\ + \frac{1-\delta_{180}{}^\nu}{16\pi(\delta_{180}-1)\delta_{180}{}^\nu} \left(3\cos^2\theta - 1 \right) , \tag{6}$$

where:

$\nu = \frac{3-\mu}{2}$, $\delta = \frac{u^2}{3(n-1)^2}$, while δ_{180} is δ calculated for angle $\theta = 180°$.

Another useful quantity for the function FF is the ratio of backward scattering b_b to the total scattering coefficient b:

$$B_p(\mu,n) = \frac{\delta_{90}{}^\nu - 1}{2(1-\delta_{90})\delta_{90}{}^\nu} , \tag{7}$$

where δ_{90} denotes the value of δ for $\theta = 90°$.

Above defined functions appear to be the best of currently known analytical phase functions of seawater. Advantageous features of the FF theory include the shape similar to the functions measured in the marine environment, a relatively simple form (equation 6), continuity, analytical integrability and the small number of parameters. An important advantage of these functions is the fact that its parameters are values associated with the optical properties of the medium. These are the hyperbolic slope of the scattering particle size distribution μ, and the refractive index of particles n.

2. PARAMETRIZATION OF THE FOURNIER-FORAND PHASE FUNCTION

While the size distributions of suspended particles can be measured using for example a Coulter counter (see [2], [18] and [11]), and measurement results can be approximated by the slope of the hyperbolic distribution μ, the determination of the effective refractive index n of suspended particles is more complicated. There are many works about indirect methods of determining the refractive index.

The effective refractive index of the particles suspended in seawater was determined from measurements of the scattering function (see [22], [4] and [12]) and its comparison with modeling results. But FF functions are used mainly when you can not make nephelometric measurements. Moreover, this method leads to significantly different results when it is based on the same scattering function but the particle size distribution was not known [10], [3] and [23].

So there is a requirement for development another method of determining the effective refractive index of marine suspensions. These include, for example, a method based on actual measurements of the inherent optical properties of seawater developed by Twardowski et al. [21]. In the literature one can find description of the refractive index obtainment of selected types of plankton cell suspensions, for example by analyzing the composition of the substance from which they are constructed [1] or by measuring selected components such as carbon and chlorophyll [20].

Due to difficulties in determining parameters n and μ of the Fournier-Forand function attempts were performed to replace it with another, easy to determine quantities. The first such attempt took Mobley et al. [13] who, considering the parameters of Petzold, proposed a simple linear relationship between n and μ. The parameter of this dependence was the relative backward scattering coefficient B_p, defined as the ratio of backscattering coefficient to total scattering coefficient. In the Fig. 1 the linear dependence $n(\mu)$ given by Mobley et al., (grey dashed line), and the parameters of the Petzold phase function (grey diamond) are shown. The diagram shows also the contours of selected B_p, related to the parameters n and μ with the equation (7). According to [13] parameters of the FF functions n and μ can be found as the intersection of dashed line $n(\mu)$ with the appropriate contour B_p.

After measurements of the volume scattering function in the Southern Baltic Sea [9], it turned out that the FF functions calculated using the Mobleys parameterization differ significantly from the measured phase functions. For this reason, Freda and Piskozub [8] proposed a new parameterization of Fournier-Forand phase function. They calculated the best-fit parameters of the function obtained experimentally. The resulting parameters are shown in the Fig. 1 as black dots. It turned out that the parameters obtained from the fit are not situated along the Mobleys line, and that their position in the diagram slightly depend on B_p (the same B_p values corresponds to significantly different pairs of n and μ coefficients). Therefore, the parameter B_p should not be the only parameter to the FF function.

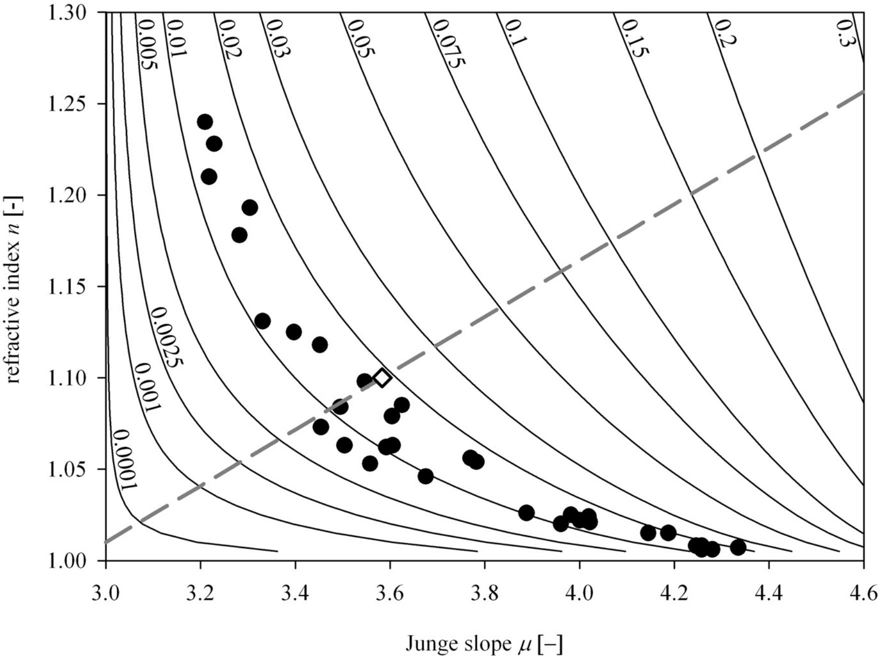

Fig. 1. The contours of the relative backward scattering coefficient B_p plotted on the diagram, which axes represents parameters *n* (vertical axis) and *μ* (horizontal axis) of the Fournier-Forand phase function. Black dots represents the parameters of best fit of the FF function to measurements made at a wavelength of 443 nm. The grey dashed line is the relationship given by Mobley et al. (2002). The grey diamond indicates the parameters of Petzold (from Freda and Piskozub 2007 by permission)

According to [8], a linear relationship proposed in [13] should be replaced with a group of linear functions with various slopes. Freda and Piskozub have linked these slopes with an absorption coefficient *a*, for four tested wavelengths separately:

$$\begin{aligned} n(443nm) &= (1{,}34 \cdot a - 0{,}36)(\mu\text{-}3) + 1 \\ n(490nm) &= (2{,}01 \cdot a - 0{,}23)(\mu\text{-}3) + 1 \\ n(555nm) &= (3{,}57 \cdot a - 0{,}15)(\mu\text{-}3) + 1 \\ n(620nm) &= (2{,}72 \cdot a - 0{,}04)(\mu\text{-}3) + 1 \end{aligned} \qquad (8)$$

In order to determine the parameters of the FF function, according to [8] one should find the solution of the system of equations (7) and (8), that is to find the intersection point of the absorption dependent linear function, with the corresponding contour of B_p.

Sokolov et al. [19] had a large database of volume scattering functions measured in coastal waters of the Black Sea. Basing on the article [8] they tested the opportunity to find a linear relationship between the ratio of $(n - 1)/(\mu - 3)$ and the absorption coefficient *a*. They could not establish such a relationship, hence they came to conclusion that the Freda and Piskozub parameterization is not universal and probably will not describe accurately enough the scattering function in water other than the Southern Baltic Sea water.

The parameterization of Freda and Piskozub [8] have not gained a popularity yet, probably because there is no simple formula for the parameters n and μ. The solution of equations (7) and (8), can be most easily obtained using numerical algorithms which finds points of intersection of curves. The aim of the paper was to provide a simple method of finding the parameters n and μ for water of the Southern Baltic.

3. RESULTS OF PARAMETRIZATION REVISION

The set of parameters B_p were replaced by its average value. They were successively B_p(443 nm) = 0,0136, B_p(490 nm) = 0,0132, B_p(555 nm) = 0,012, B_p(620 nm) = 0,0119. Curves describing the contours of B_p in the Fig. 1 were approximated by hyperbolic functions. This allow to achieve an analytical solution of the system of equations. As a result of the transformation and application of successive simplifications, the following results were obtained (for the four wavelengths λ):

$$\begin{aligned}
n(443nm) &= \frac{a(443nm)+0{,}7359}{0{,}6238\cdot a(443nm)+0{,}8187} \quad \text{and} \quad \mu = \frac{3{,}0228\cdot a(443nm)}{a(443nm)-0{,}0694} \\
n(490nm) &= \frac{a(490nm)+0{,}4384}{0{,}6311\cdot a(490nm)+0{,}4792} \quad \text{and} \quad \mu = \frac{3{,}0259\cdot a(490nm)}{a(490nm)-0{,}0402} \\
n(555nm) &= \frac{a(555nm)+0{,}2282}{0{,}6439\cdot a(555nm)+0{,}2505} \quad \text{and} \quad \mu = \frac{3{,}0278\cdot a(555nm)}{a(555nm)-0{,}0197} \\
n(620nm) &= \frac{a(620nm)+0{,}2614}{0{,}6457\cdot a(620nm)+0{,}2867} \quad \text{and} \quad \mu = \frac{3{,}0302\cdot a(620nm)}{a(620nm)-0{,}0227}
\end{aligned} \quad . \qquad (9)$$

The spectral variation of the slope of particle size distribution μ, which should not depend on the wavelength of light, results from the dependence of these values from the wavelength dependent absorption coefficient. The variability of n and μ parameters as a function of the absorption coefficient are shown in the Fig. 2.

The disadvantage of this method is limited range of absorption coefficient value, for which we obtain correct (greater than 1) values of the n parameters (refractive index). This follows from the fact that derived formulas (9) comes from averages and approximations obtained for multiple measurements. Another drawback of this parameterization method is limited in time and space character of input data. Formulas can be incorrect when one will use them for water from other basins or other seasons (because of different composition of seawater suspensions).

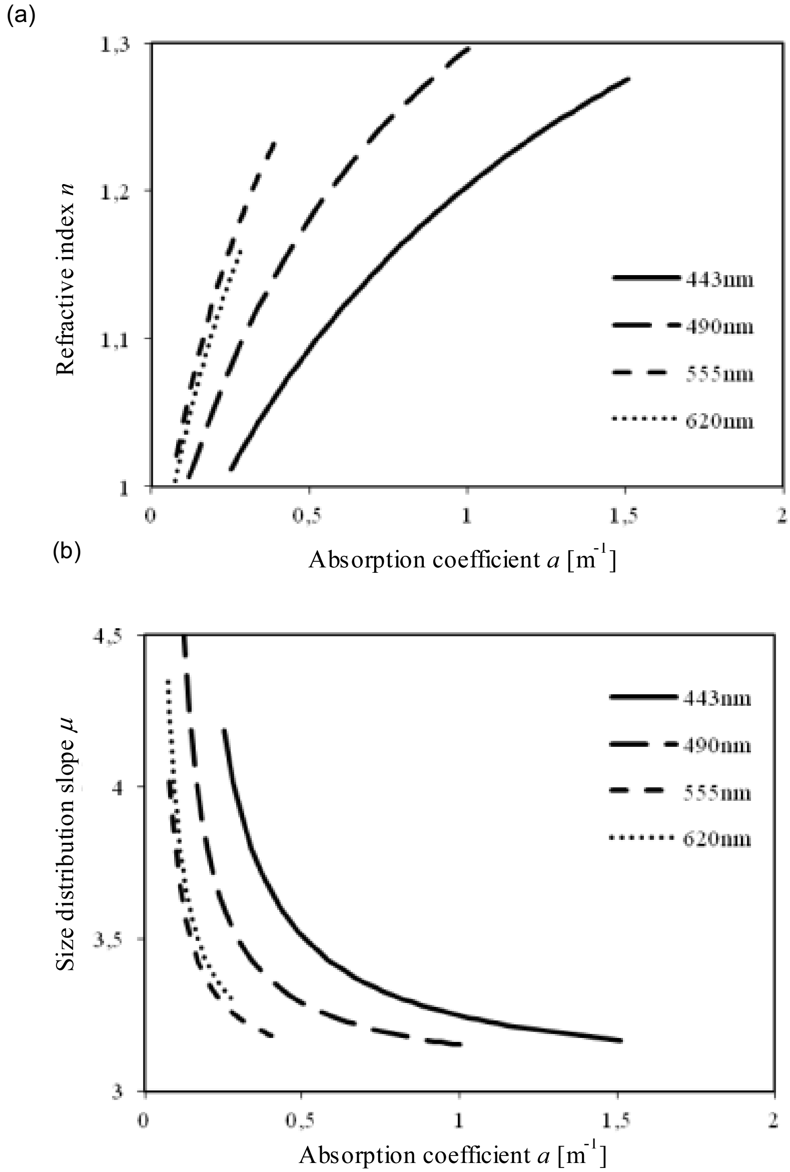

Fig. 2. The variability of parameters of the Fournier-Forand phase function *n* (a) and *μ* (b) against the absorption coefficient for four examined wavelengths

CONCLUSIONS

The set of equations (9) describes parameters of the Fournier-Forand phase function, that have been designated, basing on actual measurements of optical properties in the Southern Baltic waters. Substituting the values of absorption coefficient, obtained by measurement, to the equations (9) allows for estimating the shape of the angular characteristic of light scattering phase functions. The above mentioned method is aimed to obtain such spatial variation of the scattering function for commonly performed (in the Baltic Sea waters) measurements of the absorption coefficient.

REFERENCENS

1. Aas E., *Refractive index of phytoplankton derived from its metabolic composition*, J. Plankton Res. 1996, no. 18, p. 2223–2249.
2. Bader H., *The hyperbolic distribution of particle sizes*, J. Geophys. Res., 1970, no. 75(15), p. 2822–2830.
3. Brown O.B., Gordon H.R., *Two component Mie-scattering models of Sargasso Sea particles*, Appl. Opt., 1973, no. 12, p. 2461–2465.
4. Brown O.B., Gordon H.R., *Comment on „Method for the determination of the index of refraction of particles suspended in the ocean"*, J. Opt. Soc. Am., 1973, no. 63, p. 1616–1617.
5. Forand J.L., Fournier G.R., *Particle distributions and index of refraction estimation for Canadian Waters*, [in] *Airborne and In-water Underwater Imaging*, G. Gilbert (ed.), Proc. SPIE 3761, 1999, p. 34–44.
6. Fournier G.R., Forand J.L., *Analytic phase function for ocean water*, [in:] *Ocean Optics XII,* J.S. Jaffe (ed.), Proc. SPIE 2258, 1994, p. 194–201.
7. Fournier G.R., Jonasz M., *Computer-based underwater imaging analysis*, [in:] *Airborne and In-water Underwater Imaging,* G. Gilbert (ed.), Proc. SPIE 3761, 1999, p. 62–77.
8. Freda W., Piskozub J., *Improved method of Fournier-Forand marine phase function parameterization*, Opt. Express, 2007, no. 15, p. 12763–12768.
9. Freda W., Król T., Martynov O.V., Shybanov E.B., Hapter R., *Measurements of Scattering Function of sea water in Southern Baltic,* Eur. Phys. J. S. T., 2007, no. 144, p. 147–154.
10. Gordon H.R., Brown O.B., *A theoretical model of light scattering by Sargasso Sea particulates*, Limnol. Oceanogr., 1972, no. 17, p. 826–832.
11. Jonasz M., Fournier G., *Approximation of the size distribution of marine particles by a sum of log-normal functions*, Limnol. Oceanogr., 1996, no. 41(4), p. 744–754.
12. Kullenberg G., Berg Olsen N., *A comparison between observed and computed light scattering functions – II. Reports Inst. of Physical Oceanography*, Univ. of Copenhagen, 1972, no. 19.
13. Mobley C.D., Sundman L.K., Boss E., *Phase Function Effects on Oceanic Light Fields*, Appl. Opt., 2002, no. 41, p. 1035–1050.
14. Otremba Z., *Oil droplets as light absorbents in seawater*, Optics Express, 2007, 15(14), p. 8592–8597.
15. Otremba, Z., Piskozub, J., *Polarized phase functions in oil-in-water emulsion*, Optica Applicata, 2009, no. 39(1), p. 129–133.
16. Otremba, Z., *The impact on the reflectance in VIS of a type of crude oil film floating on the water surface*, Optics Express, 2000, no. 7(3), p. 129–134.

17. Petzold T.J., *Volume scattering functions for selected ocean waters*, SIO Ref. 72-78, Scripps Inst. Oceanogr., La Jolla, 1972, no. 79.
18. Sheldon R.W., Prakash A., Sutcliffe W.H. Jr., *The Size Distribution of Particles in the Ocean*, Limnology and Oceanography, 1972, no. 17(3), p. 327–340.
19. Sokolov A., Chami M., Dmitriev E., Khomenko G., *Parameterization of volume scattering function of coastal waters based on the statistical approach*, Opt. Express, 2010, no. 18, p. 4615–4636.
20. Stramski D., *Refractive index of planktonic cells as a measure of cellular carbon and chlophyll a content*, Deep-Sea Res. I, 1999, no. 46, p. 335–351.
21. Twardowski M.S., Boss E., Macdonald J.B., Pegau W.S. et al., *A model for estimating bulk refractive index from the optical backscattering ratio and the implications for understanding particle composition in case I and case II waters*, J. Geophys. Res., 2001, no. 104, p. 14129–14142.
22. Zaneveld J.R.V., Pak H., *Method for the determination of the index of refraction of particles suspended in the ocean*, J. Opt. Soc. Am., 1973, no. 63, p. 321–324.
23. Zaneveld J. R. V., Roach D. M., Pak H., *The determination of the index of refraction distribution of oceanic particles*, J. Geophys. Res., 1974, no. 79, p. 4091–4095.

POPRAWIONA PARAMETRYZACJA FUNKCJI FAZOWYCH FOURNIERA-FORANDA DLA WÓD BAŁTYKU POŁUDNIOWEGO

Streszczenie

Rozpraszanie i identyfikacja materiałów ropopochodnych w wodzie morskiej może być skuteczne tylko przy użyciu zdalnych metod optycznych. Pociąga to za sobą konieczność odróżnienia zanieczyszczeń od naturalnych składników wody morskiej. Stąd wynika potrzeba dokładnego poznania optycznych właściwości wód morskich. W skład rzeczywistych właściwości optycznych wód morskich wchodzą współczynniki absorpcji i rozpraszania oraz funkcja fazowa rozpraszania światła. Współczynniki absorpcji światła i rozpraszania światła opisują odpowiednio względną ilość światła, która zostaje zaabsorbowana lub zmieni kierunek propagacji na jednostkę długości prostoliniowej wiązki świetlnej. Do opisu rozpraszania światła potrzebna jest jeszcze funkcja, która zawiera informację o względnych kątowych rozkładach natężeń rozproszonego światła – funkcja fazowa. Pomiary funkcji fazowych w środowisku morskim są trudne do wykonania i stąd wykonywane są niezmiernie rzadko. Dlatego wszelkie obliczenia transmisji energii promienistej w wodach morskich wykonywane są z zastosowaniem uśrednionych wartości funkcji pochodzącej z pomiarów, bądź jednej ze znanych analitycznych postaci funkcji fazowych. Jedną z takich postaci, która dobrze dopasowuje się do wyników pomiarów, jest tzw. funkcja Fourniera-Foranda. Parametrami tej funkcji są nachylenie hiperbolicznego rozkładu rozmiarów μ *oraz względny współczynnik załamania światła cząstek rozpraszających* n*. Ponieważ wyznaczenie tych parametrów jest zadaniem problematycznym, podjęto próby zastąpienia parametrów* n *i* μ *innymi. Udaną próbę parametryzacji funkcji Fourniera-Foranda podjęli Freda i Piskozub [8], poprawiając inną parametryzację zaproponowaną przez Mobleya i in. [13]. Jednakże parametryzacja Fredy i Piskozuba polegała na odnalezieniu rozwiązania układu równań, które najłatwiej można otrzymać metodami numerycznymi. Z tego powodu w niniejszym artykule zaproponowano uproszczenie parametryzacji Fredy i Piskozuba oraz podano proste i użyteczne formuły wiążące wartości parametrów* n *i* μ *z inną, łatwiejszą do wyznaczenia wielkością – współczynnikiem absorpcji.*

Rafał Krakowski
Akademia Morska w Gdyni

WPŁYW PODWYŻSZONEJ TEMPERATURY I CIŚNIENIA PŁYNU CHŁODZĄCEGO NA PARAMETRY PRACY TŁOKOWEGO SILNIKA SPALINOWEGO I DZIAŁANIE JEGO UKŁADU CHŁODZENIA

W artykule przedstawiono bilans cieplny tłokowego silnika spalinowego. Zaprezentowano modelowe i eksperymentalne stanowiska badawcze do badań układu chłodzenia o podwyższonej temperaturze płynu chłodzącego. Na stanowisku modelowym, w wyniku przeprowadzonych badań, wyznaczono charakterystyki przebiegów temperatury i ciśnienia cieczy chłodzącej przy 0,3 MPa. Wykazano, że istnieje możliwość utrzymania założonego stałego ciśnienia w układzie i uzyskania przy tym podwyższonej temperatury cieczy, prowadzącej do zwiększenia ekonomiczności silnika. Następnie wykonano charakterystyki prędkościowe i obciążeniowe silnika 4CT90 ze standardowym i ciśnieniowym układem chłodzenia. Wyniki badań potwierdziły korzyści wynikające ze zwiększenia temperatury cieczy chłodzącej. Z przedstawionych charakterystyk wynika, że zastosowanie ciśnieniowego układu chłodzenia wpływa na mniejsze zużycie paliwa, szczególnie przy dużej prędkości obrotowej, co przyczynia się do wzrostu sprawności ogólnej silnika. Zmalała również ilość związków toksycznych w spalinach, szczególnie przy małym obciążeniu silnika, poza tlenkami azotu. Wzrost tlenków azotu oznacza, że zastosowanie ciśnieniowego układu chłodzenia wymaga użycia dodatkowego i efektywnego układu redukującego tlenki azotu.

WSTĘP

Efektywność cieczowych układów chłodzenia można zwiększyć przez zastosowanie elektronicznego sterowania pracą zespołów tych układów i podwyższenie temperatury płynu chłodzącego. Jednak w układach, w których stosowana jest ciecz chłodząca na bazie wody, wymaga to jednoczesnego zwiększenia ciśnienia w układzie chłodzenia. Dotychczasowe badania takiego układu wskazują na możliwość zwiększenia sprawności ogólnej i zmniejszenie ilości toksycznych składników w spalinach [2]. Na potrzeby niniejszego artykułu zbadano wpływ podwyższonej temperatury i ciśnienia cieczy chłodzącej na parametry pracy tłokowego silnika spalinowego.