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MATHEMATICAL MODELLING OF CONSTRUCTION OF SHIP TURNING TRAJECTORY USING AUTONOMOUS BOW THRUSTER WORK AND RESEARCH OF BOW THRUSTER CONTROL SPECIFICS

Oleksandr Kupraty

Odessa National Maritime University, Mechnikova 34, Odessa 65029, Ukraine,
ORCID: 0000-0003-3519-504X, e-mail:kupraty@ukr.net

Abstract: The article proposes a method of constructing the trajectory of a ship's turn using a control device. Mathematical modelling of the turn trajectory is performed using MS Excel in conjunction with the MATLAB environment. To construct a trajectory, a conditional centre of turn will be used, it will be guided by a control device, such as a radio buoy, or a lighthouse with pre-known coordinates. The construction of this trajectory has two features: a step course change when rotated equal to a sector step, and the accuracy of the trajectory is achieved by using the Kalman filter, simplified to the amplification factor. The bow thruster moment is calculating using the bow thruster shoulder. The angular velocity of turning depends on the linear speed, therefore, the angular velocity can be adjusted not only by changing the bow thruster mode of operation, but also by changing the linear speed of the vessel. The article provides the program code for constructing a trajectory in the MATLAB environment, which takes its initial data from MS Excel. Therefore, the work forms the basic view of autonomous bow thruster control.

Keywords: accuracy of trajectory, Kalman filter, step course change, sector step, bow thruster shoulder.

1. INTRODUCTION

To program autonomous ship control, you need to solve the problem of autonomous control of the bow thrusters and develop a method of constructing a curved trajectory, ensuring the accuracy of this trajectory. When entering the port often not enough steering gear work, is especially true for limited areas, therefore, you need to use a bow thruster. When programming an electronic control device, there is a need not only to program scenarios, but also to construct precise trajectories in narrow, winding channels.

If we approach the issue comprehensively, it is necessary to consider the efficiency of the bow thruster depending on the longitudinal component of the linear

speed of the vessel and control the main engine during the turn, focussing on the work of the bow thruster. Based on all of the above, this topic seems relevant.

2. LITERATURE ANALYSIS AND FORMULATING THE PROBLEM

In today's environment, the development of autonomous control systems is a priority in all modes of transport. For this article, the literature on the bow thruster was analysed. In particular, the works [Juditin 2010; Kopczyński 2015] are the basics for developing ship autonomous control software for turning, and describe the specifics of the bow thruster control. Work related to the Kalman filter [Cadet 2003; Maszarov 2011] was also analysed, which describes how the Kalman filter works as a powerful mathematical tool. The works [Morgas and Kopacz 2013; Botniev and Ustinov 2014; Kupraty 2020] describe the solution to direct and reverse geodesic tasks, such as the calculation of the course and distance between two points. In [Kupraty 2020], an algorithm for calculating the course between points on a rhumb line and a method of constructing the turning circle of a vessel based on the rudder angle was proposed. It contains six real variations, with the choice of the variation used in the solution depending on the size of the remainder of the divisions of longitude differences by 2π and the value of the summand of the isometric latitudes difference, R. The above documents contain the basic rules for the creation of autonomous vessels, which were adopted by the Maritime Safety Committee at its 100 and 101 sessions [IMO 2018; Interim Guidelines for Mass Trials 2019]. The work [Wieremiej 2016] is dedicated to mean square multi-purpose optimisation, including the synthesis of a course management system based on sea excitement taking into account the ship's hydrodynamic parameters, control forces and external influences.

3. METHODS AND MATERIALS

In the study of the bow thruster, the calculation-experimental method was used, as well as the Kalman filter in a simplified form, and trigonometric formulas were used to develop the software code to construct the reversal of the vessel using the bow thruster. The Kalman filter is nowadays used in DP systems.

A characteristic feature of the bow thruster is that at low linear speeds, the bow thruster turns a vessel more efficiently, and at high linear speeds, the influence factor increases, which is why a vessel should decrease her speed before using the bow thruster.

A sector step is an arc encased between two trajectory radii, the value of which is equal to the vessel step course change between the start point and the end point of this arc.

4. THE AIM AND TASKS

The aim of the work is to develop a mathematical model of the construction of a ship turning trajectory using autonomous bow thruster work control and research of bow thruster control specifics.

Tasks:

- using a known pattern of changing the influence factor of the longitudinal component of the linear speed on the efficiency of the bow thruster, construct a polynomial trend in MS Excel and identify a function in MS Excel that can be used to manage this factor numerically;
- add the bow thruster force and moment in a non-linear differential system and propose a method to control the angle speed;
- develop a program code to construct the trajectory of the ship's reversal (rotation) using the bow thruster in MATLAB.

5. RESEARCH AND ELABORATION

5.1. Change in the value of the influence factor, determining its function

To understand how the bow thruster influence factor changes, the effectiveness of the bow thruster should be illustrated under three conditions: the vessel has no movement relative to the water, low speed of the vessel, high speed of the vessel (Fig. 1 a,b,c). At zero speed, the resulting vector exits the channel of the bow thruster and the resultant force has the highest magnitude, at low speeds, the resultant force shifts to the stern and the vector decreases, and at high speeds, the resultant force comes out of the bow thruster channel, but the vector of the resulting force is decreased even more, which is evidenced by the change in the influence factor.

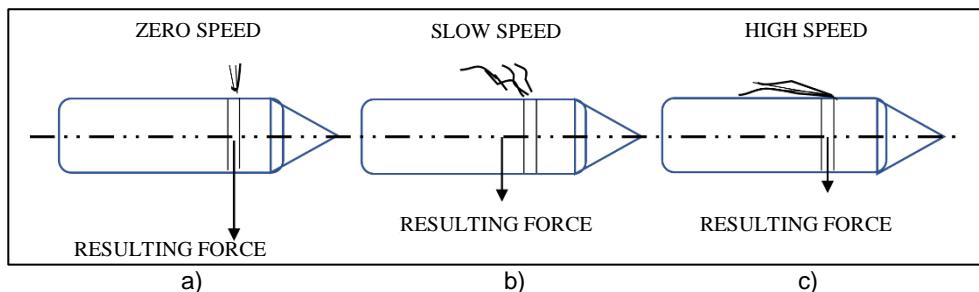


Fig. 1 a, b, c. Changing the point of the application and the value of the resulting force depending on the speed

From source [Judin 2010], you can use a pattern for the influence factor k_v , the longitudinal component of the ship's linear speed, that depends on v' , v' calculated from the relationship V_x and V_{fl} , where V_x – projection of the ship's linear velocity on the axis x , and V_{fl} – flow speed in the bow thruster channel.

$$v' = \frac{V_x}{V_{fl}} \quad (1)$$

Analytical coefficient dependence k_v from the ratio of speeds is as follows [Judin 2010]:

$$k_v = \begin{cases} 1, & \text{if } v' < 0.1 \\ 1 - 1.4v', & \text{if } 0.1 \leq v' < 0.5 \\ 0.3 + 0.3(v' - 0.5), & \text{if } 0.5 \leq v' < 1.2 \\ 1.2, & \text{if } v' \geq 1.2 \end{cases} \quad (2)$$

The influence factor depends on the speed ratio. Based on the graph below (Fig. 2), changes in the influence factor in the corresponding limits $k(v') \in [0 ; 1.2]$ and construct a polynomial to determine the function in MS Excel.

The blue line indicates changes in the influence factor within the specified boundaries, while red denotes the polynomial, which determines the function of changing the influence factor in MS Excel.

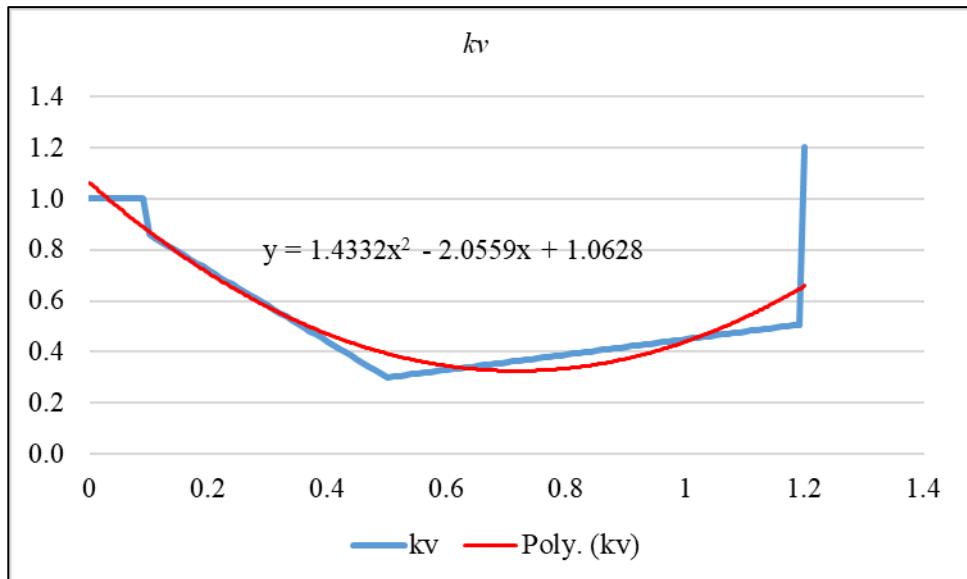


Fig. 2. Effect factor changed due to the longitudinal component of the ship's linear velocity

Therefore:

$$k(v') = 1.4332(v')^2 - 2.0559v' + 1.0628, k(v') \in [0 ; 1.2] \quad (3)$$

Based on this function, the electronic control device will be able to autonomously adjust the revs of the main engine and the bow thruster to keep the influence factor of the longitudinal component of the ship's linear velocity within the specified limits.

Therefore, the main engine will be controlled adaptively.

However, it is also especially important to say that wind forces must be calculated for effective motion control. Table 1 shows the wind force moments for m/v "UMM QARN".

Table 1. Wind force moments [kN·m]

Wind direct. [deg.]	Wind speed in [m/s]							
	5	10	15	20	25	30	35	40
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10.0	-1085	-4342	-9769	-17367	-27135	-39075	-53185	-69466
20.0	-1707	-6827	-15361	-27308	-42669	-61443	-83631	-109232
30.0	-2193	-8770	-19733	-35082	-54815	-78934	-107437	-140326
40.0	-2111	-8445	-19001	-33779	-52779	-76002	-103447	-135115
50.0	-1966	-7863	-17693	-31454	-49146	-70770	-96326	-125814
60.0	-1726	-6903	-15531	-27611	-43143	-62125	-84560	-110445
70.0	-1359	-5434	-12226	-21735	-33962	-48905	-66565	-86942
80.0	-793	-3171	-7134	-12683	-19816	-28536	-38840	-50730
90.0	-183	-731	-1644	-2922	-4566	-6575	-8950	-11689
100.0	654	2615	5884	10461	16345	23536	32035	41842
110.0	1453	5811	13075	23244	36319	52299	71185	92976
120.0	2127	8509	19144	34034	53178	76576	104229	136135
130.0	2331	9322	20976	37290	58265	83902	114200	149159
140.0	2489	9956	22402	39826	62228	89608	121967	159303

cont. Table 1

150.0	2261	9044	20349	36176	56525	81396	110788	144703
160.0	1721	6886	15493	27543	43036	61972	84350	110172
170.0	962	3849	8661	15397	24058	34643	47153	61588
180.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
190.0	-962	-3849	-8661	-15397	-24058	-34643	-47153	-61588
200.0	-1721	-6886	-15493	-27543	-43036	-61972	-84350	-110172
210.0	-2261	-9044	-20349	-36176	-56525	-81396	-110788	-144703
220.0	-2489	-9956	-22402	-39826	-62228	-89608	-121967	-159303
230.0	-2331	-9322	-20976	-37290	-58265	-83902	-114200	-149159
240.0	-2127	-8509	-19144	-34034	-53178	-76576	-104229	-136135
250.0	-1453	-5811	-13075	-23244	-36319	-52299	-71185	-92976
260.0	-654	-2615	-5884	-10461	-16345	-23536	-32035	-41842
270.0	183	731	1644	2922	4566	6575	8950	11689
280.0	793	3171	7134	12683	19816	28536	38840	50730
290.0	1359	5434	12226	21735	33962	48905	66565	86942
300.0	1726	6903	15531	27611	43143	62125	84560	110445
310.0	1966	7863	17693	31454	49146	70770	96326	125814
320.0	2111	8445	19001	33779	52779	76002	103447	135115
330.0	2193	8770	19733	35082	54815	78934	107437	140326
340.0	1707	6827	15361	27308	42669	61443	83631	109232
350.0	1085	4342	9769	17367	27135	39075	53185	69466
360.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

The bow thruster longitudinal coordinates $X_{BT} = 336$ m (from APP).

The moment of the wind force changes depending on the wind speed and direction. The bow thruster is located 336 metres from APP. This coordinate is used to calculate the necessary force via Formula 4:

$$F = \frac{|M|}{X_{BT}} ; \quad (4)$$

For example, calculation of the force to keep a vessel on course for a wind speed of 10 m/s from the wind direction 250° F is executed as follows:

$$F = \frac{|-5811\text{kN}|}{336\text{m}} \approx 17.295 \text{kN}.$$

The required power can be calculated proportionally:

$$k_T = \frac{F}{T}; \quad (5)$$

$$k_T = \frac{17.295\text{kN}}{257 \text{ kN}} \approx 0.0673$$

$$P_T = k_T \times P; \quad (6)$$

$$n_T = k_T \times n; \quad (7)$$

$$P_T = 0.0673 \times 1800 = 121.14 \text{ kwt};$$

$$n_T = 0.0673 \times 287 \approx 19.3 \text{ rpm};$$

Thus, this method can be used to calculate the necessary force of the bow thruster to hold the ship in position.

5.2. Kinematic and hydrodynamic specifics of bow thruster control during turning

Some important basics should be kept in mind when calculating the kinematics of ship turning:

Linear acceleration $a = v \cdot \omega$, $a = \omega^2 \cdot R$, because $v = \omega \cdot R$.

$$\omega = \omega_0 + \frac{1}{m} \cdot F \cdot t. \quad (8)$$

For a ship, it should be described in ideal conditions as follows:

$$\omega = \omega_0 + \frac{1}{m(1+k_{66})} \cdot F \cdot t, \quad (9)$$

$$\text{where } k_{66} = \frac{2T}{B}.$$

As was mentioned above, $\Delta\psi$ is the sector step and course change step at same time, therefore, the ship angle speed relative to the centre of turning and relative to the pivot point are the same:

$$\frac{\Delta\psi_s}{\Delta t} = \frac{\Delta\psi_{TC}}{\Delta t}. \quad (10)$$

By supplementing the basic system [10] of non-linear differential equations with the bow thrust, we get the following system:

$$\dot{V}_x = \frac{1.8T_v - X_H - X_R + m(1-k_{22})V_y\omega + F_x}{m(1+k_{11})}; \quad (11)$$

$$\dot{V}_y = \frac{Y_H + Y_R + T_B + F_y}{m(1+k_{22})}; \quad (12)$$

$$\dot{\omega} = \frac{M_H + M_R + M_B + M_z}{J_{ZZ}(1+k_{66})}; \quad (13)$$

where:

- V_x – longitudinal component of ship speed,
 - V_y – transversal component of ship speed; ($V_y = V \cdot \sin \beta$),
 - $\omega - \omega_z$ – angle speed,
 - T_v – propeller thrust,
 - T_B – bow thrust,
 - X_R – longitudinal component of rudder force,
 - Y_R – transversal component of rudder force,
 - M_R – moment of rudder force,
 - M_B – moment of bow thrust,
 - X_H – longitudinal component of hydrodynamic force,
 - Y_H – traversal component of hydrodynamic force,
 - M_H – moment of hydrodynamic force,
 - F_x, F_y, M_z – external forces and moments that act on the hull (wind and sea waves),
 - \dot{V}_x, \dot{V}_y – longitudinal and traversal components of ship acceleration,
 - $\dot{\omega}$ – angle of acceleration.
- } Steering forces;

It is a necessary step for the simulations to linearise the non-linear differential system to a linear one.

Therefore, first of all in the simulationit should be mentioned as initial data that $V_x(t) \equiv V$ (for this rpm),

$V_y(t) \equiv 0, \varphi \equiv 0, \omega_0(t) \equiv 0, \delta \equiv 0$ (rudder angle), $F_x \equiv 0, F_y \equiv 0, M_z \equiv 0$.

So it will be changed by adding steering and external forces that will effect the ship accelerations on the longitudinal and traversed axis.

The ship turning motion can be adjusted by a PID regulator, so the steering device (control device) will adjust the bow thruster force to maintain the angle speed within appropriate limits. Therefore, the proposal is to calculate the bow thruster moment by the following formula:

$$M_B = T_B \cdot L_B, \quad (14)$$

where:

- T_B – bow thrust,

L_B – shoulder of bow thruster which is measured on the longitudinal axis from the ship's centre of gravity to the bow thruster propeller.

Therefore, if $\omega_0(t) \equiv 0$, then $\omega = \dot{\omega} \cdot t$.

Thus, by complementing the system of differential equations with the bow thrust and the moment of the bow thrust, it is possible to control the angle speed of the vessel by changing the bow thrust moment in an adaptive autonomous mode. The bow thruster shoulder should be calculated automatically based on changes in the position of the centre of gravity.

5.3. Mathematical modelling of the construction of ship turning trajectory using autonomous bow thruster work

When constructing a turn trajectory (reversal) with the help of the bow thruster, it should be understood that at low speeds the efficiency of the steering gear decreases, and the efficiency of the bow thruster increases. When constructing a turn trajectory (reversal) the vessel with the help of a steering gear involves only the steering gear and the main engine or engines.

The trajectory of the ship's rotation with the help of the bow thruster should be guided by a certain geometric centre (beacon, lighthouse, elevation, underwater beaming acoustic sensor system), the coordinates of which are known. With this orientation, the vessel is safely rotated in a limited area. When you construct a trajectory, you calculate the distance between the centre of the rotation and the beginning of the turn, and this distance taken as the right radius and the first bearing from the centre of the rotation to the beginning of the turn is also calculated. At the known initial course of the ship and the course on which the vessel will be deployed, the bearing from the geometric centre to the end point of the reversal is calculated. The difference between the initial and the final bearings is equal to the difference between the initial and the final course (Fig. 3).

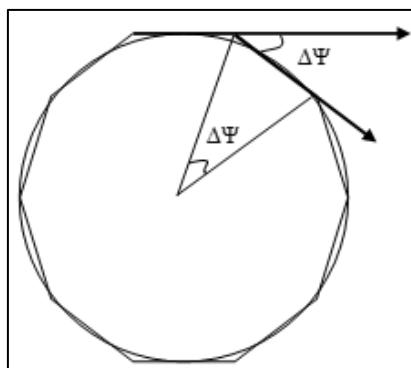


Fig. 3. Step course change and sector step

It is clearly visible in Figure 3 that the course change and the sector step between the two points are equal.

The autonomous control system will allow you to hold the necessary angular speed steady on a given trajectory.

The initial radius of the turn is accepted as correct, so subsequent radii, calculated trigonometrically, require adjustment.

The most effective tool is the Kalman filter, but in this case, it should be simplified to a gain factor. The initial form of the Kalman filter is as follows:

$$X_{k+1} = K_{k+1} \cdot X_k + \varepsilon_{k+1} \quad (15)$$

To convert ε_{k+1} to zero, $\varepsilon_{k+1} = 0$.

As the first radius is correct, the gain factor \mathbf{K} is calculated by formula (16):

$$K_{k+1} = \frac{R}{R_{k+1}}, \quad (16)$$

where $R_{k+1} \diamond R$, because it is counted on another bearing and/or to another point. R and R_{k+1} were calculated as great circle distance.

Figure 4 shows an example of such a turnaround. Effective work of the bow thruster is provided at slow speed. The radius of the turning circle doesn't change and doesn't depend on the linear or angle speed. The angle speed changes proportionally to the linear speed. Under normal conditions, rotation should be provided by the steering device at speed 2.5 knots. In normal practice, the bow thruster is ineffective at a ship speed greater than 3 knots.

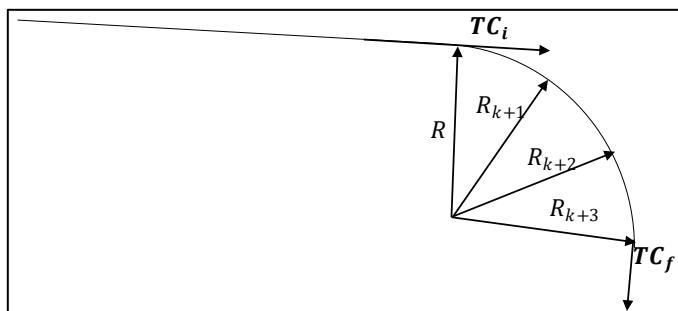


Fig. 4. Geometry of the ship's rotation trajectory

The bearing changes at the same time as the change of course. Since the radii and coordinates of the circle points are calculated trigonometrically, it is necessary to adjust $\Delta\varphi_{k+1}$ by the appropriate gain factor \mathbf{K} .

$$\begin{aligned} \varphi_{k+1} &= \varphi_0 + K_{k+1} \times R \times \cos PL_{k+1} \\ \varphi_{k+1} &= \varphi_0 + K_{k+1} \times \Delta\varphi_{k+1} \end{aligned} \quad (17)$$

This adjustment can be depicted as a matrix:

$$\begin{vmatrix} \Delta\varphi_1 \\ \Delta\varphi_2 \\ \Delta\varphi_3 \\ \Delta\varphi_4 \\ \Delta\varphi_5 \\ \Delta\varphi_6 \\ \Delta\varphi_7 \end{vmatrix} = \begin{vmatrix} \Delta\varphi_{1k+1} \\ \Delta\varphi_{2k+1} \\ \Delta\varphi_{3k+1} \\ \Delta\varphi_{4k+1} \\ \Delta\varphi_{5k+1} \\ \Delta\varphi_{6k+1} \\ \Delta\varphi_{7k+1} \end{vmatrix} \times \begin{vmatrix} K_{1k+1} \\ K_{2k+1} \\ K_{3k+1} \\ K_{4k+1} \\ K_{5k+1} \\ K_{6k+1} \\ K_{7k+1} \end{vmatrix} \quad \text{or} \quad \begin{vmatrix} \Delta\varphi_1 \\ \Delta\varphi_2 \\ \Delta\varphi_3 \\ \Delta\varphi_4 \\ \Delta\varphi_5 \\ \Delta\varphi_6 \\ \Delta\varphi_7 \end{vmatrix} = \begin{vmatrix} \Delta\varphi_{1k+1} \\ \Delta\varphi_{2k+1} \\ \Delta\varphi_{3k+1} \\ \Delta\varphi_{4k+1} \\ \Delta\varphi_{5k+1} \\ \Delta\varphi_{6k+1} \\ \Delta\varphi_{7k+1} \end{vmatrix} \times \begin{vmatrix} R/R_{1k+1} \\ R/R_{2k+1} \\ R/R_{3k+1} \\ R/R_{4k+1} \\ R/R_{5k+1} \\ R/R_{6k+1} \\ R/R_{7k+1} \end{vmatrix} \quad (18)$$

And the final type of calculation of the matrix of geographical coordinates takes the following form:

$$\begin{vmatrix} \varphi_{1k+1} \\ \varphi_{2k+1} \\ \varphi_{3k+1} \\ \varphi_{4k+1} \\ \varphi_{5k+1} \\ \varphi_{6k+1} \\ \varphi_{7k+1} \end{vmatrix} = \begin{vmatrix} \varphi_0 \\ \varphi_0 \\ \varphi_0 \\ \varphi_0 \\ \varphi_0 \\ \varphi_0 \\ \varphi_0 \end{vmatrix} + \begin{vmatrix} K_{1k+1} \\ K_{2k+1} \\ K_{3k+1} \\ K_{4k+1} \\ K_{5k+1} \\ K_{6k+1} \\ K_{7k+1} \end{vmatrix} \times \begin{vmatrix} R \\ R \\ R \\ R \\ R \\ R \\ R \end{vmatrix} \times \begin{vmatrix} \cos PL_{1k+1} \\ \cos PL_{2k+1} \\ \cos PL_{3k+1} \\ \cos PL_{4k+1} \\ \cos PL_{5k+1} \\ \cos PL_{6k+1} \\ \cos PL_{7k+1} \end{vmatrix} \quad (19)$$

The geographic coordinates are filtered by the gain factor K_{k+1} in such a way.

Figure 4 shows the trajectory of the vessel, in which the initial course is 95° , and the final is 186° . At the same time, the difference between the initial and the final bearing from the centre of the reversal of the vessel is equal to the difference between the initial and the final course. The bearings were calculated in MS Excel using the algorithm outlined in the source [Kupraty 2020].

The sector step is calculated in MS Excel by the following formula:

$$SS = \frac{\Delta PL}{INT(\Delta PL/SS_i)} \quad (20)$$

MATLAB program code is given in Appendix A.

Example 1

Construction of a starboard trajectory

Finish course	186
Initial course	59.5
	40.85083333
Coordinates of the initial point	29.26383333

Coordinates of the geometric centre of the rotation	40.84483333 29.26950000
Initial radius (distance from the centre of turning to the initial point)	0.4432
Sector step	8.272727273
PERMIT (“TURN” in MORSE CODE)	1 001 010 10
Board (port side – “0”, starboard side – “1”)	1
First bearing	324.46

The word “TURN” in Morse code is used for the permit code.

Calculation matrix of bearings **PL** in MS Excel based on the turning board:

$$\mathbf{PL} = [324.46 \quad 332.89 \quad 341.33 \quad 349.76 \quad 358.20 \quad 6.63 \quad 15.06 \quad 23.50 \quad 31.93 \\ 40.37 \quad 48.80 \quad 57.24 \quad 65.67 \quad 74.11 \quad 82.54 \quad 90.98]$$

$$\mathbf{K} = [1.0132 \quad 1.0073 \quad 1.0027 \quad 0.9996 \quad 0.9983 \quad 0.9988 \quad 1.0012 \quad 1.0052 \quad 1.0106 \\ 1.0169 \quad 1.0237 \quad 1.0304 \quad 1.0363 \quad 1.0409 \quad 1.0438 \quad 1.0446]$$

Based on all of the above data, MATLAB calculates the coordinates of the turning matrix.

$$\mathbf{TM} = \begin{vmatrix} 40.85083333 & 40.85145723 & 40.85185038 & 40.85209979 & 40.85220390 & 40.85216217 & 40.85197481 \\ 29.26383333 & 29.26521662 & 29.26650419 & 29.26784194 & 29.26920668 & 29.27057614 & 29.27192820 \\ 40.85164288 & 40.85116874 & 40.85055675 & 40.84981421 & 40.84895230 & 40.84798692 & \dots & \dots & 40.84470182 \\ 29.27324016 & 29.27448806 & 29.27564634 & 29.27668801 & 29.27758536 & 29.27831142 & \dots & \dots & 29.27924605 \end{vmatrix}$$

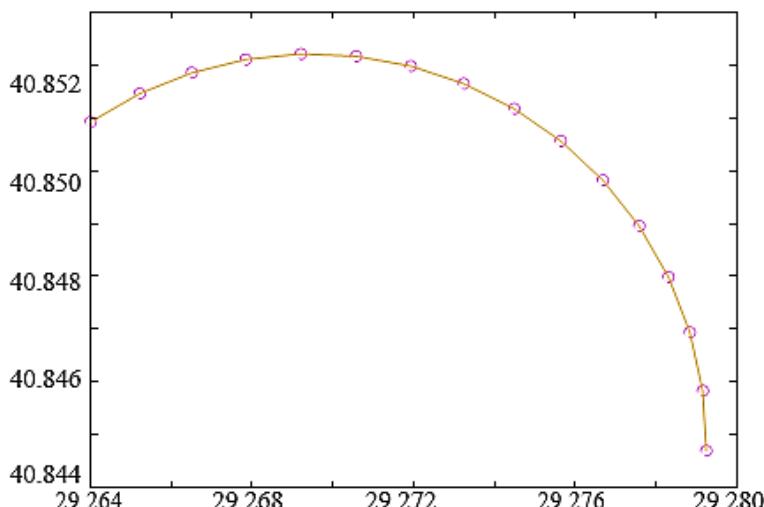


Fig. 5. Trajectory simulation of a starboard turn in MATLAB



Fig. 6. Illustration of a starboard turn in OpenCPN 5.2.4 (Tuzla Shipyard Entrance)

Example 2

Constructing of a port side trajectory

Finish course	90
Initial course	81.1
Coordinates of the initial point	40.83791525 29.22261913
Coordinates of the geometric centre of the rotation	40.84855000 29.21783333
Initial radius (distance from the turning centre to the initial point)	0.6753
Sector step	8.165116279
PERMIT (“TURN” in MORSE CODE)	1 001 010 10
Board (port side – “0”, starboard side – “1”)	0
First bearing	161.20

Figure 7 shows the port side turning trajectory, the appropriate “PERMIT” and the board of turning calculated in MS Excel.

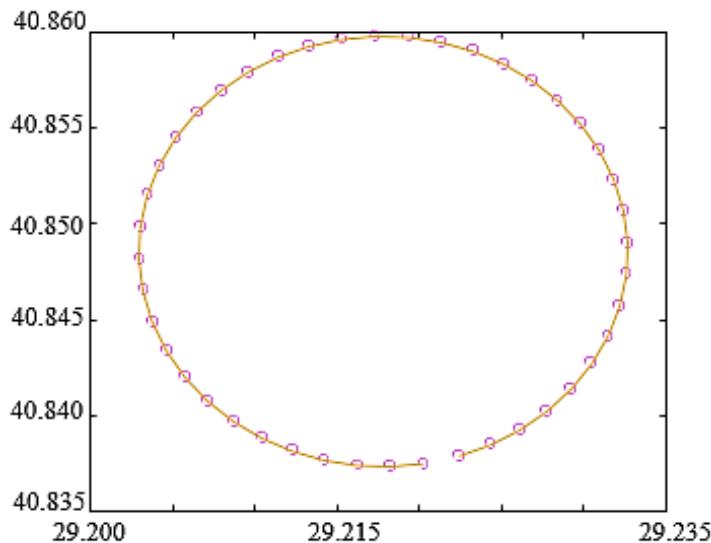


Fig. 7. Simulation of the port side trajectory in MATLAB

The bearing matrix for this rotation is as follows:

$$\text{PL} = [161.20 \quad 153.03 \quad 144.87 \quad 136.70 \quad 128.54 \quad 120.37 \quad 112.21 \quad 104.04 \quad 95.88 \\ 87.71 \quad 79.55 \quad 71.38 \quad 63.22 \quad 55.05 \quad 46.89 \quad 38.72 \quad 30.56 \quad 22.39 \\ 14.23 \quad 6.06 \quad 357.90 \quad 349.73 \quad 341.57 \quad 333.40 \quad 325.24 \quad 317.07 \quad 308.91 \\ 300.74 \quad 292.58 \quad 284.41 \quad 276.25 \quad 268.08 \quad 259.92 \quad 251.75 \quad 243.59 \quad 235.42 \\ 227.26 \quad 219.09 \quad 210.93 \quad 202.76 \quad 194.59 \quad 186.43 \quad 178.26 \quad 170.10]$$

$$\text{K} = [1.0027 \quad 1.0072 \quad 1.0129 \quad 1.0193 \quad 1.0259 \quad 1.0322 \quad 1.0376 \quad 1.0417 \quad 1.0441 \\ 1.0446 \quad 1.0430 \quad 1.0396 \quad 1.0347 \quad 1.0287 \quad 1.0222 \quad 1.0156 \quad 1.0096 \quad 1.0046 \\ 1.0008 \quad 0.9987 \quad 0.9983 \quad 0.9996 \quad 1.0026 \quad 1.0070 \quad 1.0126 \quad 1.0190 \quad 1.0256 \\ 1.0319 \quad 1.0374 \quad 1.0416 \quad 1.0440 \quad 1.0446 \quad 1.0431 \quad 1.0398 \quad 1.0349 \quad 1.0290 \\ 1.0225 \quad 1.0159 \quad 1.0099 \quad 1.0048 \quad 1.0010 \quad 0.9988 \quad 0.9982 \quad 0.9995]$$

The bearings are changed to a corresponding sector step, which is calculated according to Formula 20, and the rotation matrix is calculated according to the bearings:

$$\text{TM} = \begin{vmatrix} 40.83791525 & 40.83844657 & \dots & 40.83746892 \\ 29.22261913 & 29.22432621 & \dots & 29.22027627 \end{vmatrix}$$

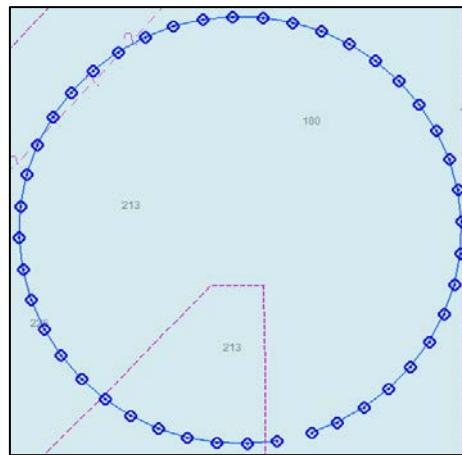


Fig. 8. Illustration of port side turning in OpenCPN 5.2.4

Thus, the code is correct for both starboard side rotation and port side rotation, constructing a trajectory as a set of points.

If analysing the gain factor, it can be noted that even the first coefficients in the matrix are not equal to one. This is because, when the coordinates of the initial point are calculated, there is a margin of error, which is eliminated by the Kalman filter gain factor. The Kalman filter in this case is used in a simplified form, but this does not affect the accuracy of the trajectory in any way. The developed code can be used to simulate the trajectory of the vessel using adaptive control of the bow thruster and the ship's engine. In determining the gain factors, the distance between the initial point of movement of the vessel, the coordinates of which are known, and the conditional centre of rotation, the coordinates of which are also known, is taken. This distance is used as the right radius of rotation, which minimises the error.

6. CONCLUSIONS

A polynomial function is proposed to control the bow thruster autonomously, which can be used to control the value of the longitudinal factor of the linear velocity. Therefore, the steering will be controlled adaptively, adjusting the angular velocity in such a way as to maintain the radius of rotation. To construct the trajectory of the ship, a program code for data processing and charting the trajectory of the rotation has been written in MATLAB.

The proposed method of autonomous regulation of the ship's angular velocity allows the control device to be programmed according to the position of the centre of gravity and the maximum power of the bow thruster. Using the proposed method of calculating the moment of thrust of the bow thruster, the bow thruster revs can be adjusted by checking the mathematical model with a real angular acceleration.

The Kalman filter gain factor is used to filter the coordinates of the trajectory points. Filtering is done by calculating the initial radius between the starting point and the centre of rotation, the coordinates of which are known. Calculating the radius for the first bearing, it was noticed that the coordinates of the calculated starting point are different from their observatory values; this difference was reflected in the coefficients 1.0132 and 1.0027 for the first and second example, respectively. This means that the proposed method will provide sufficient accuracy of the rotational trajectory of the ship. The proposed code can also be used in a variety of autonomous systems where accuracy of the rotational trajectory must be provided.

The main scientific contribution of this work is to offer methods of constructing the trajectory of the ship's rotational movement and autonomous control of the steering device, which in turn will allow in the future the control device of a large vessel to be programmed to the fourth level of autonomy.

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APPENDIX A

```
Latc=xlsread('Rotation_with_bow_thruster','OBRACANIE','C9');
Longc=xlsread('Rotation_with_bow_thruster','OBRACANIE','C10');
Lats=xlsread('Rotation_with_bow_thruster','OBRACANIE','C7');
Longs=xlsread('Rotation_with_bow_thruster','OBRACANIE','C8');
% xlsread() is a function
Rpl=3443.918467; % Earth radius in nm for WGS 84

e=sqrt(2/298.257223563-(1/298.257223563)^2)% eccentricity for
WGS 84

R=Rpl*(acos(cos((90-Latc).*pi/180).*cos((90-
Lats).*pi/180)+sin((90-Latc).*pi/180).*sin((90-
Lats).*pi/180).*cos((Longc-Longs).*pi/180))); % calculation of
distances (radii)

PL=xlsread('Rotation_with_bow_thruster','OBRACANIE','C16:DR16');
% bearing matrix
y1=Latc+R.*cos(PL.*pi/180)./60;
x1=Longc -(7915.70447.*log10(tan((45+Latc/2).*pi/180).*((1-
e*sin(Latc*pi/180)./(1+e*sin(Latc*pi/180)))))-7915.70447
.*log10(tan((45+y1./2).*pi/180).*((1-e*sin(y1.*pi/180)
./(1+e*sin(y1.*pi/180))))).*tan(PL.*pi/180)./60;
R1 = Rpl*(acos(cos((90-Latc).*pi/180).*cos((90-
y1).*pi/180)+sin((90-Latc) .*pi/180).*sin((90-
y1).*pi/180).*cos((Longc-x1).*pi/180)));
% calculation of trajectory as a coordinate matrix without
filtration by gain rate
K=R./R1; % gain rate coefficient

y=Latc+K.*R.*cos(PL.*pi/180)./60;
x=Longc -(7915.70447.*log10(tan((45+Latc/2).*pi/180).*((1-
e*sin(Latc*pi/180)./(1+e*sin(Latc*pi/180)))))-7915.70447.*
log10(tan((45+y./2).*pi/180).*((1-e*sin(y.*pi/180./ (1+e*sin(y.*
pi/180))))).* tan(PL.*pi/180)./60;
% calculation of trajectory as a coordinate matrix with
filtration by gain rate
z=[x y];
for i =1:length(x)
plot (x(i), y(i), 'mo');
hold on
pause (0.01);
end
plot (x,y);
hold off % plotting command
```